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# Information and maximum power in a feedback controlled Brownian ratchet

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**Abstract.** Closed-loop or feedback controlled ratchets are Brownian motors that operate using information about the state of the system. For these ratchets, we compute the power output and we investigate its relation with the information used in the feedback control. We get analytical expressions for one-particle and few-particle flashing ratchets, and we find that the maximum power output has an upper bound proportional to the information. In addition, we show that the increase of the power output that results from changing the optimal open-loop ratchet to a closed-loop ratchet also has an upper bound that is linear in the information.

**PACS.** 05.40.-a Fluctuation phenomena, random processes, noise, and Brownian motion – 89.70.+c Information theory and communication theory – 02.30.Yy Control theory

## 1 Introduction

Brownian ratchets have been studied in different contexts due to their theoretical importance in non-equilibrium statistical mechanics and their potential relevance for applications in disciplines like nanotechnology, condensed matter or biology [1–4]. Many studies deal with the performance of these devices (see Refs. [1,5] for comprehensive reviews) concentrating on open-loop ratchets, as those obtained fluctuating an uniform external force (rocking ratchets [6,7]), or an external asymmetric potential (flashing ratchets [6,8]), either randomly or periodically. On the other hand, closed-loop or feedback controlled ratchets, as the so-called instant maximization protocol [9] and the threshold protocol [10], use information of the state of the system to operate. The feedback ratchet of [9] has been recently proposed as an effective model to describe the stepping motion of the two-headed kinesin [3]. Other 'information-dependent' rectification mechanism have been recently proposed to model certain chemical and biological systems [4].

The previous works [9–11] about closed-loop ratchets focused on the study of the flux and its maximization. In particular, it has been shown that the increase of the flux performance when the optimal open-loop control is changed to a closed-loop control has an upper bound proportional to the square root of the information used by the controller [11].

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In this paper, we consider another measure of the performance, viz. the power output, with the aim of getting further insight in the relation between information and the increase of performance in a system with thermal fluctuations. We oppose to the flux a constant load force [12] in order to compute the potential energy gain by the particles thanks to the action of the controller. The generalization of the methods developed in [11] allow us to obtain the relations between the maximum power output and the information that the controller uses.

## 2 The model

The collective feedback ratchet that we investigate has two basic ingredients, namely, $N$ Brownian particles and a controller. The controller acts on the particles switching on and off a potential $V(x)$ according to the control policy and to the information received about the state of Brownian particles through a noisy channel.

Specifically, we consider $N$ overdamped Brownian particles at temperature $T$ in a piecewise linear saw-tooth potential

$$V(x) = \begin{cases} \frac{xV_0}{aL} & \text{if } 0 \le \frac{x}{L} \le a, \\ V_0 - \frac{V_0}{1-a}\left(\frac{x}{L} - a\right) & \text{if } a < \frac{x}{L} \le 1, \end{cases} \qquad (1)$$

of height $V_0$, asymmetry parameter $a$, and period $L$, i.e. $V(x) = V(x + L)$. The potential is switched on and off according to the instant maximization of the center-of-mass velocity protocol (see Ref. [9]), which switches on the potential only when the net force due to the potential on the particles would be positive. In order to obtain work from the system operation we oppose to the flow of particles an homogeneous static force $F_{\rm ext}$; thus, the total force acting on the particles when the potential is on is $F_{\rm tot}(x) = F(x) - F_{\rm ext}$, with $F(x) = -V'(x)$, and $F_{\rm tot}(x) = -F_{\rm ext}$ when the potential is off. The state of the system is described by the positions $x_i(t)$ of the particles that satisfy the Langevin equations

$$\gamma \dot{x}_i(t) = \alpha(t) F(x_i(t)) - F_{\rm ext} + \xi_i(t); \quad i = 1, \ldots, N, \quad (2)$$

where $\gamma$ is the friction coefficient (related to the diffusion coefficient $D$ through Einstein's relation $D = k_B T/\gamma$) and $\xi_i(t)$ are Gaussian white noises of zero mean and variance $\langle \xi_i(t) \xi_j(t') \rangle = 2\gamma k_B T \delta_{ij} \delta(t - t')$. The dichotomous function $\alpha(t)$ [$\alpha = 0$ (potential off) or $\alpha = 1$ (potential on)] implements the action of the controller. The control policy uses the information received from the system through a noisy channel that we model with a binary symmetric channel [13]. This channel passes the sign of the net force

$$f(t) = \frac{1}{N} \sum_{i=1}^{N} F(x_i(t)) \quad (3)$$

to the controller with an error probability $p$ known as the noise level of the channel, so when $f(t) > 0$ ($< 0$) the controller switches on (off) the potential with probability $1 - p$. Therefore, the feedback protocol and the noisy channel lead to the effective control policy

$$\alpha_{\rm eff}(t) = (1 - p)\Theta(f) + p\Theta(-f), \quad (4)$$

with $\Theta$ the Heaviside function [$\Theta(x) = 1$ if $x > 0$, else $\Theta(x) = 0$]. This effective control policy is equivalent to the protocol of instant maximization through a noisy channel provided many measurement and control actions are performed in the characteristic time of the system evolution, which is the case we consider here.

Our aim is to study the dependence of the maximum power output with the information. On one hand, the average information transmitted through the noisy channel is quantified in terms of the mutual information [13] that the controller gets from the state of the system. Our case — the noisy measurement of the sign of the net force — is equivalent to a noisy channel called the binary symmetric channel in information theory. For this case the mutual information can be computed (see Sect. 8.1.4 of [13]), and it is given (in bits) by

$$I = H(q) - H(p), \quad (5)$$

with $H(x) = -x \log_2 x - (1 - x) \log_2 (1 - x)$, $q = (1 - p)b + p(1 - b)$ the probability that the controller receives a negative sign, and $b$ the probability that the actual sign of the net force is negative. Therefore, the information $I$ that the controller gets about the system is greatly determined by the noise level $p$ of the channel; the maximum information is reached for $p = 0$ and it is at most 1 bit, while for $p = 1/2$ the channel becomes completely random and no information of the system is received by the controller. When the probability $b$ does not depend on $p$, the relation between the noise level $p$ and the information $I$ can be easily expanded around $p = 1/2$ and reads

$$I(p) = \frac{1}{\ln 2} \sum_{k \text{ even}} \frac{2^k}{k(k-1)} \left[1 - (1 - 2b)^k\right] \left(p - \frac{1}{2}\right)^k. \quad (6)$$

Inverting this relation to leading order we get for $p < 1/2$ the result [11]

$$p \simeq \frac{1}{2} - \sqrt{\frac{I \ln 2}{8b(1-b)}}. \quad (7)$$

On the other hand, a positive power output is obtained when there is a net flux against the load $F_{\rm ext}$ that tilts the potential. In the stationary regime the center-of-mass moves with a mean velocity $\langle \dot{x}_{\rm cm} \rangle$ and then the average power output (work obtained per unit time) is given by

$$P = F_{\rm ext} \langle \dot{x}_{\rm cm} \rangle. \quad (8)$$

We first analyze the dependence of the power output with the information for the case of one particle and later for the few-particle ratchet.

## 3 One particle

We start with the one particle case ($N = 1$) where an effective potential that includes the effects of the load, the control protocol, and the binary symmetric channel can be constructed. The system dynamics can be viewed as the result of the action of the periodic effective force $F_{\rm eff}(x) = \alpha_{\rm eff}(x)F(x) - F_{\rm ext}$ that derives from an effective potential. Using units $L = 1$ and $k_B T = 1$, this effective potential can be written in terms of $K := pV_0 + F_{\rm ext} a$ and $M := (1 - p)V_0 - F_{\rm ext}(1 - a)$ as

$$V_{\rm eff}(x) = \begin{cases} \frac{xK}{a} & \text{if } 0 \le x \le a, \\ K - M\frac{x-a}{1-a} & \text{if } a < x \le 1 \end{cases} \quad (9)$$

in the interval $[0, 1]$, and outside $V_{\rm eff}(x) = V_{\rm eff}(y) + (x - y)V_{\rm eff}(1)$, with $y \equiv x \mod 1$, $y \in [0, 1]$. Equation (9) and Figure 1 show that the effect of increasing the noise level $p$ is to diminish in the effective potential the average tilt that induces a positive flux, while the effect of increasing the load $F_{\rm ext}$ is to tilt the effective potential opposing the positive flux.

Solving the stationary Fokker-Planck equation for this effective potential the stationary mean velocity for one particle is obtained (in units $L = 1$ and $k_B T = 1$):

$$\langle \dot{x} \rangle = \frac{DK^2 M^2 A}{AE - B^+ B^-}, \quad (10)$$

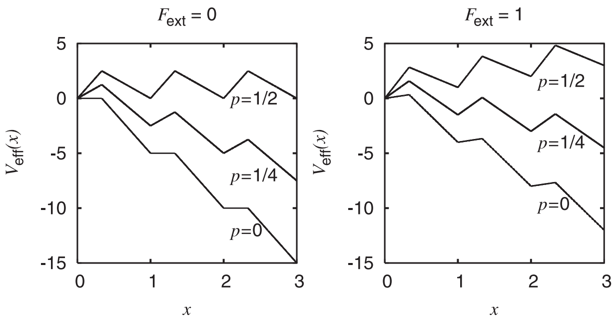


**Fig. 1.** Effective potentials for one particle and noise levels $p = 0$, $1/4$, and $1/2$ with $V_0 = 5$ and $a = 1/3$. Left panel without external load, and right panel with external load $F_{\rm ext} = 1$. Units: $L = 1$, $D = 1$, and $k_BT = 1$. Note that for $p = 1/2$ the effective potential is equal to $V(x)/2 + F_{\rm ext}x$.

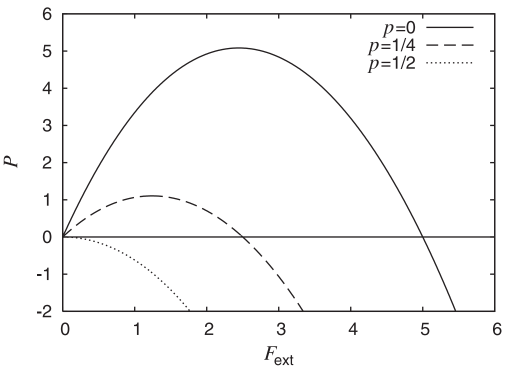


**Fig. 2.** Power output versus the load for $V_0 = 5$ and $a = 1/3$ in the one particle case [Eqs. (8) and (10)]. Units: $L = 1$, $D = 1$, $k_BT = 1$.

with

$$\begin{aligned}
A &:= 1 - e^{K-M}, \\
B^{\pm} &:= [aM + (1-a)K]e^{\pm K} \\
&\quad - (1-a)Ke^{\pm(K-M)} - aM, \\
E &:= a^2M^2(1 - K - e^{-K}) \\
&\quad + a(1-a)KM(1-e^{M})(1-e^{-K}) \\
&\quad + (1-a)^2K^2(1 + M - e^{M}).
\end{aligned}$$

For $p < 1/2$, there is a positive current for forces smaller than the "stopping force" $F_{\rm stop}$ (the one that leads to the cancellation of the velocity), so a work is done against the load for $F_{\rm ext} \in (0, F_{\rm stop})$. For $p = 1/2$, the stopping force is zero, because no positive flux is obtained even in the absence of the external load. Our noisy control acts instantaneously, i.e. in a time scale much faster than the characteristic times of the system [$(aL)^2/(2D)$ for the diffusion time and $\gamma(1-a)^2L^2/V_0$ for the characteristic time of the drift induced by the potential]. Thus, for $p = 1/2$ the potential $V(x)$ is randomly switched on and off very fast and the particle just feels the average potential. This implies that the effective potential in absence of the load, namely $V(x)/2$ (see Fig. 1), is not tilted, giving a zero flux for the $p = 1/2$ case for zero load. Therefore, in order to get work the noise level of the channel should be $p \in [0, 1/2)$ and the value of $F_{\rm stop}$ is obtained equating equation (10) to zero,

$$F_{\rm stop} = \frac{V_0}{L}(1 - 2p). \tag{11}$$

Substituting equation (10) in equation (8) we get the analytical expression for the power output in the one-particle ratchet. The dependence with the load is plotted in Figure 2 for noise levels $p = 0$, $p = 1/4$, and $p = 1/2$. The positive regions correspond to the system doing work against the external force. The load $F^*_{\rm ext}$ that maximizes the power output lies between 0 and $F_{\rm stop}$ and it is given by the condition

$$\frac{\partial P}{\partial F_{\rm ext}}(F^*_{\rm ext}) = 0. \tag{12}$$

In general, it is a function of the noise level of the binary symmetric channel and it also depends on the physical parameters of the potential, $V_0$ and $a$. The condition (12) gives a transcendental equation for $F^*_{\rm ext}$ that can be numerically solved in order to obtain the maximum power output,

$$P_{\max} = P(F^*_{\rm ext}). \tag{13}$$

This equation gives the dependence of the maximum power with the noise level $p$, which is related with the information $I$ through equation (5). This last equation requires to compute $b$, which can be obtained integrating over the space interval $[0, aL]$ the stationary distribution of the Fokker-Planck equation for the effective potential (9),

$$b = \langle \dot{x} \rangle \left(\frac{a}{K}\right)^2 \left\{ \left(1 - e^{-K}\right) \left[1 + \frac{1 - e^{-K} + (1-a)(1-e^{-M})K/(aM)}{e^{-K} - e^{-M}}\right] - K \right\} \tag{14}$$

(units $L = 1$, $D = 1$, $k_BT = 1$). Therefore, the combination of equations (5), and (12–14) permits to obtain the (implicit) exact dependence of the maximum power developed by the Brownian motor as a function of the information gathered by the controller (see Fig. 3).

We analyze now the regime of small potentials in the one particle case. For small potentials ($V_0 \lesssim k_BT$) the value of the external force that maximizes the power is also small [remember Eq. (11) and the fact that $F^*_{\rm ext} < F_{\rm stop}$]. In this regime, the velocity (10) reduces to $\langle \dot{x} \rangle \simeq D(M - K)$, or, recovering units,

$$\langle \dot{x} \rangle \simeq (1 - 2p)\frac{V_0}{\gamma L} - \frac{F_{\rm ext}}{\gamma}. \tag{15}$$

We see that there are two contributions to the velocity: the current effect due to the white thermal noise and the control through the binary channel, $(1 - 2p)V_0/(\gamma L)$, and the net drift due to the load, $-F_{\rm ext}/\gamma$. We highlight that for small potentials (and loads) these two effects appear

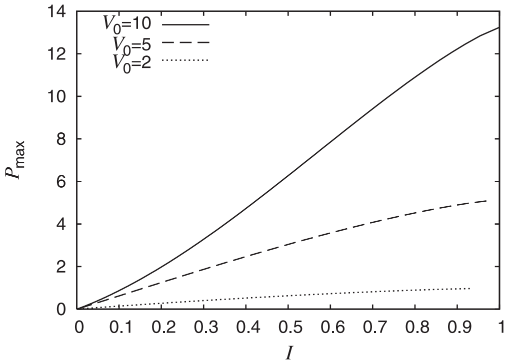


**Fig. 3.** Maximum power output in the one particle case as a function of the information used. The curves are for heights of the potential $V_0 = 2, 5, 10$ and asymmetry parameter $a = 1/3$. Units: $L = 1$, $D = 1$, $k_BT = 1$.

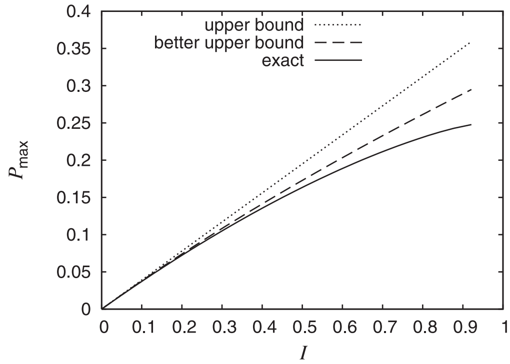


**Fig. 4.** Maximum power output in the one particle case as a function of the information for $V_0 = 1$ and $a = 1/3$, and comparison with the upper bound (18) and the better upper bound (20). Units: $L = 1$, $D = 1$, $k_BT = 1$.

uncoupled, and the result is independent of the asymmetry of the potential. This independency of the asymmetry for small potentials can be understood realizing that in this case the effective potential is well approximated by a flat potential with the same average slope, i.e., $V_{\rm eff}(x) \simeq [-V_0(1-2p)/L + F_{\rm ext}]x$.

Applying equation (12) to the power output computed using equation (15) we obtain

$$F^*_{\rm ext} = \frac{V_0}{2L}(1-2p) = \frac{F_{\rm stop}}{2}, \tag{16}$$

and then the power output is

$$P_{\max} = \frac{F^{*2}_{\rm ext}}{\gamma} = \frac{V_0^2}{4\gamma L^2}(1-2p)^2. \tag{17}$$

On the other hand, for small potential heights $b \simeq a$, and using equation (7) we get

$$P_{\max} \simeq R_1 I, \tag{18}$$

with $R_1$ a constant that depends on the physical parameters of the system,

$$R_1 = \frac{V_0^2 \ln 2}{8\gamma L^2 a(1-a)}. \tag{19}$$

Notice that the dependence on the asymmetry $a$ does appear here because it determines the relation between $p$ and $I$ [Eq. (7)], as $b \simeq a$ for small potentials.

Therefore, equation (17) indicates that for small potential heights and small values of the information (i.e., $p \sim 1/2$) the maximum power is approximately directly proportional to the information gathered. In addition, we have numerically checked that equation (18) gives an upper bound of the maximum power for any potential height $V_0$ and for any value of the information $I$.

A better approximation for the dependence of the maximum power output with the information can be found using the result of inverting equation (6) up to fourth order,

$$P_{\max} \simeq S_1(-1+\sqrt{1+S_2 I}), \tag{20}$$

with

$$S_1 = \frac{3V_0^2}{4\gamma L^2}\frac{1-(1-2a)^2}{1-(1-2a)^4};\ S_2 = \frac{4}{3}\frac{1-(1-2a)^4}{[1-(1-2a)^2]^2}\ln 2, \tag{21}$$

which is also an upper bound of $P_{\max}$ for any potential and information values. In Figure 4 these upper bounds [Eqs. (18) and (20)] are compared with the exact result for $V_0 = k_BT$.

## 4 Few particles

Let us now study a collective ratchet composed of a few particles and show that the results are similar to those in the one particle case. Summing and averaging the Langevin equations (2), the average velocity of the center-of-mass in the stationary state can be written as

$$\gamma\langle \dot{x}_{\rm cm}\rangle = \langle \alpha_{\rm eff}(f) f\rangle - F_{\rm ext}. \tag{22}$$

An approximate solution can be found assuming, as in [9,11], that: (i) the position of the particles are statistically independent, and (ii) the probability of finding a particle in a negative force interval (for example $[0, aL]$) is $a$. These assumptions are verified for small potentials and small loads even in the presence of noise, and they imply that the probability distribution for $f$ is approximately Gaussian,

$$\rho(f) \simeq \frac{1}{\sqrt{2\pi\Sigma^2}}\, e^{-f^2/(2\Sigma^2)}, \tag{23}$$

with $\Sigma$ the amplitude of the fluctuations of the net force, given by $\Sigma = \frac{V_0}{L\sqrt{a(1-a)N}}$ (see Ref. [9]). Following [11], we get for the center-of-mass velocity

$$\langle \dot{x}_{\rm cm}\rangle \simeq \frac{\Sigma}{\gamma\sqrt{2\pi}}(1-2p) - \frac{F_{\rm ext}}{\gamma}. \tag{24}$$

This result is the sum of the center-of-mass velocity without the external load [11] plus the drift $-F_{\rm ext}/\gamma$ due to the external load. We see that, like in the one particle case, these two effects are decoupled for small potential heights. Expression (24) agrees with the results of numerical simulations of the stochastic evolution equations (2).

Applying equations (8) and (12), it can be shown that the maximum power is reached for

$$F^*_{\mathrm{ext}} = \frac{V_0}{L\sqrt{8\pi a(1-a)N}}(1-2p) \tag{25}$$

and takes the value

$$P_{\max} = \frac{V_0^2}{\gamma L^2 8\pi a(1-a)N}(1-2p)^2, \tag{26}$$

or simply $P_{\max} = F^{*2}_{\mathrm{ext}}/\gamma$. Therefore, we also have in the regime of few particles (and small potentials) that the maximum power is at first approximation directly proportional to the information,

$$P_{\max} \simeq R_N I. \tag{27}$$

In the previous expression, the constant $R_N$ depends only on the physical parameters of the system, in particular the number of particles $N$,

$$R_N = \frac{V_0^2 \ln 2}{\gamma L^2 16\pi a(1-a)b(1-b)N}, \tag{28}$$

where $b$ can be calculated for small potentials and loads,

$$b = \sum_{n>aN}^{N} \binom{N}{n} a^n (1-a)^{N-n}, \tag{29}$$

using the same assumptions that lead to the Gaussian approximation for $\rho(f)$. We have checked numerically that equation (27) is an upper bound for the maximum power output in the few particles case. Again, as in the one-particle ratchet, a linear upper bound has been found for the maximum power output that the system can get using a certain amount of information $I$.

## 5 Correlation

The previous expression of the different physical magnitudes of the system in terms of the noise level $p$ can be recast in terms of a correlation $C$ that we introduce in this section. The main underlying idea is that the presence of noise in the control induces a decorrelation between the relevant magnitudes of the control policy. In the instant maximization of the center-of-mass protocol [9] the switching of the potential only depends on the sign of the net force, namely $\mathrm{sgn}f$, and the presence of noise in the control implies that the controller does not use the actual value $\mathrm{sgn}f$ but a value $\mathrm{sgn}\tilde{f}$. The correlation between these quantities,

$$C = \langle \mathrm{sgn}f \; \mathrm{sgn}\tilde{f} \rangle, \tag{30}$$

can be written as

$$C = P_{++} + P_{--} - P_{+-} - P_{-+}, \tag{31}$$

where $P_{+-}$ is the join probability of having $\mathrm{sgn}f = +1$ while the controller receives $\mathrm{sgn}\tilde{f} = -1$, and analogously for the other joint probabilities. As in our system $\mathrm{sgn}\tilde{f}$ is different from $\mathrm{sgn}f$ with probability $p$ (the noise level) these joint probabilities can be easily computed by noting that $P_{-+} = bp$, $P_{--} = b(1-p)$, $P_{+-} = (1-b)p$ and $P_{++} = (1-b)(1-p)$, with $b$ the probability of $\mathrm{sgn}f$ being $-1$. Therefore, the correlation can be parameterized in terms of the noise level $p$ as

$$C = 1 - 2p. \tag{32}$$

In other words, the effect of the noise is to decrease the correlation $C$, which has its maximum value ($C = 1$) for zero noise and its minimum ($C = 0$) for a completely noisy policy, $p = 1/2$.

Finally, using equation (32), the relations derived in previous sections can be restated in terms of the correlation. For example, equations (17) and (26) reads

$$P_{\max} = \frac{V_0^2 C^2}{4\gamma L^2}, \tag{33}$$

(one particle case), and

$$P_{\max} = \frac{V_0^2 C^2}{\gamma L^2 8\pi a(1-a)N}, \tag{34}$$

(few particles case).

This reformulation helps to understand the physical meaning of the relations derived in the previous sections giving a complementary view. In addition, it indicates that the noisy control considered can give an effective description of other feedback ratchets with an imperfect operation of the feedback control. For instance, this effective description has been shown to be valid in time-delayed feedback ratchets consisting of few particles [14], where $f = f(t)$, $\tilde{f} = f(t-\tau)$ (with $\tau$ being the time-delay), and the correlation can be computed just from the time series of the net force $f(t)$.

## 6 Comparison with open-loop protocols

The instantaneous maximization of the center-of-mass velocity is the optimal protocol to maximize the power output in the one particle case for a noiseless channel ($p = 0$). Thus, we also expect this protocol to give a power output close to the maximum possible value in the few particles case and in the presence of noise with a memoryless protocol (note that protocols with memory can perform error corrections). Therefore, we expect equations (18) and (27) to be upper bounds of the maximum power output that can be obtained with a memoryless closed-loop protocol that uses an amount of information $I$ about the system.

In addition, the maximum power output obtained with open-loop protocols is much smaller than that obtained with efficient closed-loop protocols. For instance, for the saw-tooth potential with parameters $V_0 = 5k_BT$

and $a = 1/3$, the periodic protocol with optimum periods $\mathcal{T}_{\mathrm{on}} \simeq 0.06L^2/D$ and $\mathcal{T}_{\mathrm{off}} \simeq 0.04L^2/D$ gives a small maximum power $P_{\mathrm{max}}^{\mathrm{open}} \simeq 0.04V_0^2/(\gamma L^2)$, which is reached for a load $F_{\mathrm{ext}}^* \simeq 0.25V_0/L$. In contrast, the closed-loop one-particle ratchet yields a maximum power $P_{\mathrm{max}}^{\mathrm{closed}} \simeq 5.1V_0^2/(\gamma L^2)$ for $F_{\mathrm{ext}}^* \simeq 2.4V_0/L$ when it works without noise in the channel. Therefore, the linear equations (18) and (27) are also good estimates of the maximum improvement that can be attained changing from the optimal open-loop control to a closed-loop protocol, i.e.,

$$P_{\mathrm{max}}^{\mathrm{closed}} - P_{\mathrm{max}}^{\mathrm{open}} \leq RI, \tag{35}$$

where $R$ is a constant depending on the system's characteristics; see equations (19) and (28).

## 7 Concluding remarks

In this article we have analyzed the relation between the information about the state of the system used by the controller and the power output in a feedback controlled ratchet. We have obtained exact analytic results for one-particle ratchets, and also approximate simple expressions for the maximum power output in both one-particle and few-particle ratchets. Moreover, we have found that the increase of the maximum power output when we change from the optimal open-loop protocol to a closed-loop protocol has an upper bound proportional to the information used by the controller. Also an upper bound proportional to the information was found in [15] for the entropy reduction in a general closed-loop controlled system. The result obtained in the present paper for the maximum power output is the analog upper bound of the one found in [11] for the flux, but with the important difference that the upper bound for the flux was proportional to the square root of the information.

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## References


1. P. Reimann, Phys. Rep. **361**, 57 (2002)
2. H. Linke, Appl. Phys. A **75**, 167 (2002); P. Hänggi, F. Marchesoni, F. Nori, Ann. Phys. **14**, 51 (2005)
3. M. Bier, Biosystems **88**, 301 (2007)
4. V. Serreli, C.-F. Lee, E.R. Ray, D. Leigh, Nature **445**, 523 (2007); E.A. Kay, D.A. Leigh, F. Zerbetto, Angew. Chem. Int. Ed. **46**, 72 (2007)
5. J.M.R. Parrondo, B.J. Cisneros, Appl. Phys. A **75**, 179 (2002); H. Linke, M.T. Downton, M.J. Zuckermann, Chaos **15**, 026111 (2005)
6. R.D. Astumian, M. Bier, Phys. Rev. Lett. **72**, 1766 (1994)
7. M.O. Magnasco, Phys. Rev. Lett. **71**, 1477 (1993); H. Kamegawa, T. Hondou, F. Takagi, Phys. Rev. Lett. **80**, 5251 (1998); L. Machura, M. Kostur, P. Talkner, J. Łuczka, F. Marchesoni, P. Hänggi, Phys. Rev. E **70**, 061105 (2004)
8. A. Ajdari, J. Prost, C.R. Acad. Sci. Paris II **315**, 1635 (1993); D. Suzuki, T. Munakata, Phys. Rev. E **68**, 021906 (2003)
9. F.J. Cao, L. Dinis, J.M.R. Parrondo, Phys. Rev. Lett. **93**, 040603 (2004)
10. L. Dinis, J.M.R. Parrondo, F.J. Cao, Europhys. Lett. **71**, 536 (2005); M. Feito, F.J. Cao, Phys. Rev. E **74**, 041109 (2006)
11. F.J. Cao, M. Feito, H. Touchette, *Information and flux in a feedback controlled Brownian ratchet*, preprint `arXiv:cond-mat/0703492` (2007)
12. K. Sekimoto, J. Phys. Soc. Jpn **66**, 1234 (1997); J.M.R. Parrondo, J.M. Blanco, F.J. Cao, R. Brito, Europhys. Lett. **43**, 248 (1998); A. Parmeggiani, F. Jülicher, A. Ajdari, J. Prost, Phys. Rev. E **60**, 2127 (1999)
13. T.M. Cover, J.A. Thomas, *Elements of Information Theory* (John Wiley, New York, 1991)
14. M. Feito, F.J. Cao, *Time-Delayed Feedback control of a flashing ratchet*, preprint `arXiv:0706.1496` (2007)
15. H. Touchette, S. Lloyd, Phys. Rev. Lett. **84**, 1156 (2000); H. Touchette, S. Lloyd, Physica A **331**, 140 (2004)